EXAM I
CALCULUS AB
SECTION I PART A
Time–55 minutes
Number of questions–28

A CALCULATOR MAY NOT BE USED ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(2) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1}x = \arcsin x$).

1. If $f'(x) = \ln(x-2)$, then the graph of $y = f(x)$ is decreasing if and only if

(A) $2 < x < 3$ (B) $0 < x$ (C) $0 < x < 1$ (D) $x > 1$ (E) $x > 2$

Ans

2. For $x \neq 0$, the slope of the tangent to $y = x\cos x$ equals zero whenever

(A) $\tan x = -x$

(B) $\tan x = \frac{1}{x}$

(C) $\tan x = x$

(D) $\sin x = x$

(E) $\cos x = x$

Ans

3. The function F is defined by

$F(x) = G[x + G(x)]$

where the graph of the function G is shown at the right.

The approximate value of $F'(1)$ is

(A) $\frac{7}{3}$

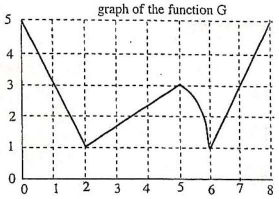
(B) $\frac{2}{3}$

(C) -2

(D) -1

(E) $-\frac{2}{3}$

graph of the function G



Ans

4. $\int_2^6 \left(\frac{1}{x} + 2x\right) dx =$

(A) $\ln 4 + 32$

(B) $\ln 3 + 40$

(C) $\ln 3 + 32$

(D) $\ln 4 + 40$

(E) $\ln 12 + 32$

Ans

5. A relative maximum of the function $f(x) = \frac{(\ln x)^2}{x}$ occurs at

(A) 0

(B) 1

(C) 2

(D) e

(E) e^2

Ans

6. Use a right-hand Riemann sum with 4 equal subdivisions to approximate the integral $\int_{-1}^{3} |2x-3|\, dx$.

(A) 13

(B) 10

(C) 8.5

(D) 8

(E) 6

Ans

7. An equation of the line tangent to the graph of $y = x^3 + 3x^2 + 2$ at its point of inflection is

(A) $y = -3x + 1$

(B) $y = -3x - 7$

(C) $y = x + 5$

(D) $y = 3x + 1$

(E) $y = 3x + 7$

Ans

8. $\int \cos(3-2x)\, dx =$

(A) $\sin(3-2x) + C$

(B) $-\sin(3-2x) + C$

(C) $\frac{1}{2}\sin(3-2x) + C$

(D) $-\frac{1}{2}\sin(3-2x) + C$

(E) $-\frac{1}{5}\sin(3-2x) + C$

Ans

9. What is $\lim_{x\to\infty} \frac{\sqrt{9x^2+2}}{4x+3}$?

(A) $\frac{3}{2}$ (B) $\frac{3}{4}$ (C) $\frac{\sqrt{2}}{3}$ (D) 1 (E) The limit does not exist.

Ans

10. Let the first quadrant region enclosed by the graph of $y = \frac{1}{x}$ and the lines $x = 1$ and $x = 4$ be the base of a solid. If cross sections perpendicular to the x-axis are semicircles, the volume of the solid is

(A) $\frac{3\pi}{64}$

(B) $\frac{3\pi}{32}$

(C) $\frac{3\pi}{16}$

(D) $\frac{3\pi}{8}$

(E) $\frac{3\pi}{4}$

Ans

11. Let $f(x) = \ln x + e^{-x}$. Which of the following is TRUE at $x = 1$?

(A) f is increasing

(B) f is decreasing

(C) f is discontinuous

(D) f has a relative minimum

(E) f has a relative maximum

Ans

12. Suppose $F(x) = \int_0^{x^2} \frac{1}{2+t^3}\,dt$ for all real x, then $F'(-1) =$

(A) 2 (B) 1 C) $\frac{1}{3}$ (D) -2 (E) $-\frac{2}{3}$

Ans

13. What is the average (mean) value of $2t^3 - 3t^2 + 4$ over the interval $-1 \le t \le 1$?

(A) 0

(B) $\frac{7}{4}$

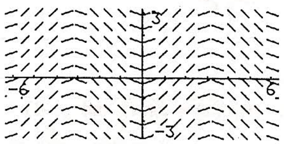
(C) 3

(D) 4

(E) 6

Ans

14. The slope field for a differential equation $\frac{dy}{dx} = f(x,y)$ is given in the figure. The slope field corresponds to which of the following differential equations?



(A) $\frac{dy}{dx} = \tan x \cdot \sec x$

(B) $\frac{dy}{dx} = \sin x$

(C) $\frac{dy}{dx} = \cos x$

(D) $\frac{dy}{dx} = -\sin x$

(E) $\frac{dy}{dx} = -\cos x$

Ans

15. What is $\lim_{x\to 1} \frac{\sqrt{x}-1}{x-1}$?

(A) 0

(B) $\frac{1}{2}$

(C) 1

(D) $\frac{3}{2}$

(E) The limit does not exist.

Ans

16. If $y = \cos^2 x - \sin^2 x$, then $y' =$

(A) -1

(B) 0

(C) $-2(\cos x + \sin x)$

(D) $2(\cos x + \sin x)$

(E) $-4(\cos x)(\sin x)$

Ans

17. The area under the graph of $y = 4x^3 + 6x - \frac{1}{x}$ on the interval $1 \le x \le 2$ is

(A) $32 - \ln 2$

(B) $30 - \ln 2$

(C) $24 - \ln 2$

(D) $\frac{99}{4}$

(E) 21

Ans

24. The acceleration at time $t > 0$ of a particle moving along the x-axis is $a(t) = 3t + 2$ ft/sec^2. If at $t = 1$ seconds the velocity is 4 ft/sec and the position is $x = 6$ feet, then at $t = 2$ seconds the position $x(t)$ is

(A) 8 ft (B) 11 ft (C) 12 ft (D) 13 ft (E) 15 ft

Ans

25. The approximate value of $y = \sqrt{3 + e^x}$ at $x = 0.08$, obtained from the tangent to the graph at $x = 0$, is

(A) 2.01

(B) 2.02

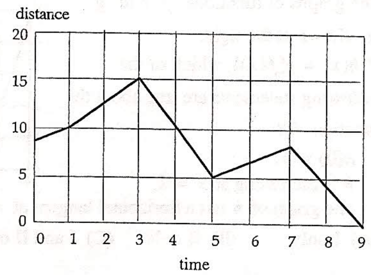
(C) 2.03

(D) 2.04

(E) 2.05

Ans

26. A leaf falls from a tree into a swirling wind. The graph at the right shows the vertical distance (feet) above the ground plotted against time (seconds).



According to the graph, in what time interval is the speed of the leaf the greatest?

(A) $1 < t < 3$

(B) $3 < t < 5$

(C) $5 < t < 7$

(D) $7 < t < 9$

(E) none of these

Ans

27. Water is flowing into a spherical tank with 6 foot radius at the constant rate of 30π cu ft per hour. When the water is h feet deep, the volume of water in the tank is given by

$$V = \frac{\pi h^2}{3}(18 - h).$$

What is the rate at which the depth of the water in the tank is increasing at the moment when the water is 2 feet deep?

(A) 0.5 ft per hr

(B) 1.0 ft per hr

(C) 1.5 ft per hr

(D) 2.0 ft per hr

(E) 2.5 ft per hr

Ans

28. The graph of the function $f(x) = 2x^{5/3} - 5x^{2/3}$ is increasing on which of the following intervals.

I. $1 < x$ II. $0 < x < 1$ III. $x < 0$

(A) I only (B) II only (C) III only (D) I and II only (E) I and III only

Ans

EXAM I
CALCULUS AB
SECTION I PART B
Time–50 minutes
Number of questions–17

A GRAPHING CALCULATOR IS REQUIRED FOR SOME QUESTIONS ON THIS PART OF THE EXAMINATION

Directions: Solve each of the following problems, using the available space for scratchwork. After examining the form of the choices, decide which is the best of the choices given and fill in the box. Do not spend too much time on any one problem.

In this test:

(1) The exact numerical value of the correct answer does not always appear among the choices given. When this happens, select from among the choices the number that best approximates the exact numerical value.

(2) Unless otherwise specified, the domain of a function f is assumed to be the set of all real numbers x for which $f(x)$ is a real number.

(3) The inverse of a trigonometric function f may be indicated using the inverse function notation f^{-1} or with the prefix "arc" (e.g., $\sin^{-1} x = \arcsin x$).

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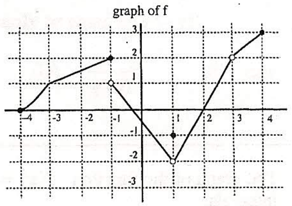
1. The function f is defined on the interval [–4, 4] and its graph is shown to the right. Which of the following statements are true?



I. $\lim_{x\to 1} f(x) = -1$

II. $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} = 2$

III. $\lim_{x\to -1^+} f(x) = f(-3)$

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

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2. For $f(x) = \sin^2 x$ and $g(x) = 0.5x^2$ on the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, the instantaneous rate of change of f is greater than the instantaneous rate of change of g for which value of x?

(A) −0.8 (B) 0 (C) 0.9 (D) 1.2 (E) 1.5

Ans

3. If $f(x) = 2x^2 - x^3$ and $g(x) = x^2 - 2x$, for what values of a and b is $\int_a^b f(x)\,dx > \int_a^b g(x)\,dx$?

I. $a = -1$ and $b = 0$ II. $a = 0$ and $b = 2$ III. $a = 2$ and $b = 3$

(A) I only
(B) II only
(C) I and II only
(D) I and III only
(E) I, II, III

Ans

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4. If $y^2 - 3x = 7$, then $\frac{d^2y}{dx^2} =$

(A) $\frac{-6}{7y^3}$ (B) $\frac{-3}{y^3}$ (C) 3 (D) $\frac{3}{2y}$ (E) $\frac{-9}{4y^3}$

Ans

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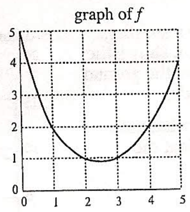
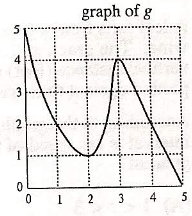
5. The graphs of functions f and g are shown at the right. If $h(x) = g[f(x)]$, which of the following statements are true about the function h?





I. $h(0) = 4$.
II. h is increasing at $x = 2$.
III. The graph of h has a horizontal tangent at $x = 4$.

(A) I only (B) II only (C) I and II only (D) II and III only (E) I, II, III

Ans

6. The minimum distance from the origin to the curve $y = e^x$ is

(A) 0.72 (B) 0.74 (C) 0.76 (D) 0.78 (E) 0.80

Ans

7. The area of the first quadrant region bounded by the y-axis, the line $y = 4 - x$ and the graph of $y = x - \cos x$ is approximately

(A) 4.50 (B) 4.54 (C) 4.56 (D) 4.58 (E) 5.00

Ans

8. The number of inflection points for the graph of $y = 2x + \cos(x^2)$ in the interval $0 \le x \le 5$ is

(A) 6 (B) 7 (C) 8 (D) 9 (E) 10

Ans

9. The rate at which ice is melting in a pond is given by $\dfrac{dV}{dt} = \sqrt{1 + 2^t}$, where V is the volume of ice in cubic feet, and t is the time in minutes. What amount of ice has melted in the first 5 minutes?

(A) 14.49 ft^3 (B) 14.51 ft^3 (C) 14.53 ft^3 (D) 14.55 ft^3 (E) 14.57 ft^3

Ans

10. The region shaded in the figure at the right is rotated about the x-axis. Using the Trapezoid Rule with 5 equal subdivisions, the approximate volume of the resulting solid is

(A) 23

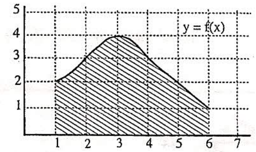
(B) 47

(C) 127

(D) 254

(E) 400

graph of $y = f(x)$



Ans

11. A particle moves along the x-axis so that at time $t \ge 0$, its position is given by $x(t) = (t + 1)(t - 3)^3$. For what values of t is the velocity of the particle increasing?

(A) all t (B) $0 < t < 1$ C) $0 < t < 3$ (D) $1 < t < 3$ E) $t < 1$ or $t > 3$

Ans

12. Let $f(x) = \dfrac{\ln e^{2x}}{x-1}$ for $x > 1$. If g is the inverse of f, then $g'(3) =$

(A) 2 (B) 1 (C) 0 (D) -1 (E) -2

Ans

13. $\displaystyle\int \frac{e^{x^2} - 2x}{e^{x^2}}\,dx$

(A) $x - e^{x^2} + C$

(B) $x - e^{-x^2} + C$

(C) $x + e^{-x^2} + C$

(D) $-e^{x^2} + C$

(E) $e^{-x^2} + C$

Ans

14. How many critical points does the function $f(x) = (x+2)^5(x^2-1)^4$ have?

(A) 2 (B) 3 (C) 4 D) 5 (E) 9

Ans

15. Let m and b be real numbers and let the function f be defined by

$$f(x) = \begin{cases} 1 + 3bx + 2x^2 & \text{for } x \le 1 \\ mx + b & \text{for } x > 1. \end{cases}$$

If f is both continuous and differentiable at $x = 1$, then

(A) $m = 1,\ b = 1$

(B) $m = 1,\ b = -1$

(C) $m = -1,\ b = 1$

(D) $m = -1,\ b = -1$

(E) none of the above

Ans

16. Suppose a car is moving with increasing speed according to the following table.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	30	36	40	48	54	60

The closest approximation of the distance traveled in the first 10 seconds is

(A) 150 ft

(B) 250 ft

(C) 350 ft

(D) 450 ft

(E) 550 ft

Ans

17. Consider the function F defined so that $F(x) + 5 = \int_2^x \sin\left(\frac{\pi t}{4}\right) dt$.

The value of $F(2) + F'(2)$ is

(A) 0

(B) 1

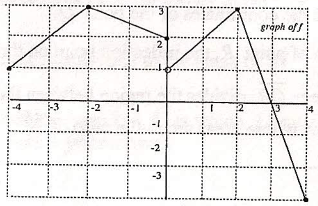
(C) $\frac{\pi}{4}$

(D) 4

(E) -4

Ans

1. Two functions, f and g, are defined on the closed interval $-4 \le x \le 4$. A graph of the function f is given in the following figure.



The table below contains some values of the differentiable function g.

x	-4	-3	-2	-1	0	1	2	3	4
$g(x)$	10	9	5	-1	0	2	6	0	-3

(a) Find $f'(3)$.

(b) Use data from the table to find an approximation for $g'(0)$. Show your work.

(c) If the function h is defined by $h(x) = g[f(x)]$, evaluate: i) $h(2)$ and ii) $h'(3)$

(d) Approximate $\int_0^4 f(x)\, dx$

18. $\int \frac{x-2}{x-1}\,dx =$

(A) $-\ln|x-1|+C$

(B) $x+\ln|x-1|+C$

(C) $x-\ln|x-1|+C$

(D) $x+\sqrt{x-1}+C$

(E) $x-\sqrt{x-1}+C$

Ans

19. Suppose that g is a function with the following two properties: $g(-x)=g(x)$ for all x, and $g'(a)$ exists. Which of the following must necessarily be equal to $g'(-a)$?

(A) $g'(a)$ (B) $-g'(a)$ (C) $\frac{1}{g'(a)}$ (D) $-\frac{1}{g'(a)}$ (E) none

Ans

20. An equation for a tangent to the graph of $y = \text{Arctan}\,\frac{x}{3}$ at the origin is:

(A) $x-3y=0$

(B) $x-y=0$

(C) $x=0$

(D) $y=0$

(E) $3x-y=0$

Ans

21. If $f(x)=\begin{cases} x^2+4 & \text{for } 0\le x\le 1 \\ 6-x & \text{elsewhere} \end{cases}$ then $\int_0^3 f(x)\,dx$ is a number between

(A) 0 and 5

(B) 5 and 10

(C) 10 and 15

(D) 15 and 20

(E) 20 and 25

Ans

22. $\frac{d}{dx}\left(\ln e^{3x}\right) =$

(A) 1

(B) 3

(C) $3x$

(D) $\frac{1}{e^{3x}}$

(E) $\frac{3}{e^{3x}}$

Ans

23. If $g'(x)=2g(x)$ and $g(-1)=1$, then $g(x)=$

(A) e^{2x}

(B) e^{-x}

(C) e^{x+1}

(D) e^{2x+2}

(E) e^{2x-2}

Ans

2. Let f be the function given by $f(x) = x^3 + 3x^2 - x + 2$.

(a) The tangent to the graph of f at the point $P = (-2, 8)$ intersects the graph of f again at the point Q. Find the coordinates of the point Q.

(b) Find the coordinates of point R, the inflection point on the graph of f.

(c) Show that the segment $\overline{QR}$ divides the region between the graph of f and its tangent at P into two regions whose areas are in the ratio of $\frac{16}{11}$.

3. Consider the graphs of $y = 3x + c$ and $y^2 = 6x$, where c is a real constant.

(a) Determine all values of c for which the graphs intersect in two distinct points.

(b) Suppose $c = -\frac{3}{2}$. Find the area of the region enclosed by the two curves.

(c) Suppose $c = 0$. Find the volume of the solid formed when the region bounded by $y = 3x$ and $y^2 = 6x$ is revolved about the x-axis.

A CALCULATOR MAY **NOT** BE USED ON THIS PART OF THE EXAMINATION.
DURING THE TIMED PORTION FOR PART B, YOU MAY GO BACK AND CONTINUE TO WORK ON THE PROBLEMS IN PART A WITHOUT THE USE OF A CALCULATOR.

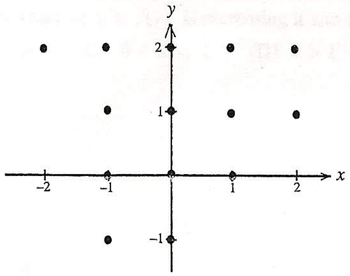
4. Consider the differential equation $\frac{dy}{dx} = x - y$.

(a) On the axes provided, sketch a slope field for the given differential equation at the fourteen points indicated.

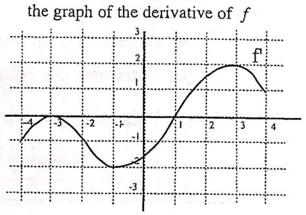
(b) Sketch the solution curve that contains the point (–1, 1).

(c) Find an equation for the straight line solution through the point (1, 0).

(d) Show that if C is a constant, then $y = x - 1 + Ce^{-x}$ is a solution of the differential equation.



5. Let f be a function defined on the closed interval $-4 \le x \le 4$. The graph of f', the **derivative** of f, is shown in the figure.



(a) Find an equation of the line tangent to the graph of f at the point (3, 1).

(b) Find all values of x on the open (–4, 4) at which f has a local minimum? Justify your answer.

(c) Estimate $f''(2)$.

(d) Find the x-coordinate of each point of inflection of the graph of f on the open interval (–4, 4). Justify your answer.

(e) At what values of x does f achieve its maximum on the closed interval [0, 4]?

6. Let A be the area of the region in the first quadrant under the graph of $y = \cos x$ and above the line $y = k$ for $0 \le k \le 1$.

(a) Determine A in terms of k.

(b) Determine the value of A when $k = \frac{1}{2}$.

(c) If the line $y = k$ is moving upward at the rate of $\frac{1}{\pi}$ units per minute, at what rate is the area, A, changing when $k = \frac{1}{2}$?